

# Tolerance Analysis Tools Take the Guesswork out of a Colorful Illumination Design Problem

LightTools illumination design and analysis software predicts process performance and production conformity using Monte Carlo tolerance analysis

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## Introduction

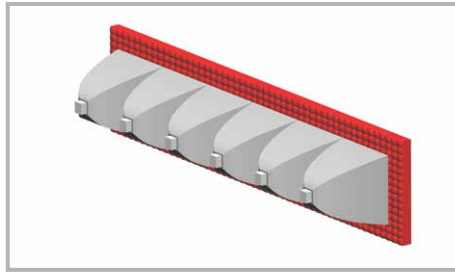
The automotive industry makes a lot of cars—73 million passenger cars worldwide in 2017 alone—and the industry loves using light-emitting diodes (LEDs) in those cars. If you've purchased a new car in the last 10 years, chances are there are LED lights on it somewhere. LEDs can be used inside and out, from the dashboard to the map lights and from the headlights to the tail lamps. The lights on the outside of a car are some of the most highly-regulated lamps that illumination designers ever encounter, and while it's an accomplishment to create a design that meets specifications for the nominal case, who ever made a nominal reflector, lens, or LED? If your design goes into production, you have to build thousands or millions of them and they all must be legal.

Tolerance analysis is a tool that's been in every lens designer's toolbox for decades. Who wouldn't want to know if they have a tolerance problem before they go into production? Sadly, tolerance analysis has largely been out of reach for illumination designers because of the amount of time needed to perform the thousands of simulations required to get a meaningful statistical tolerance result, and because of a lack of built-in tolerance analysis tools for illumination design and analysis software. But as time marches on, computers are becoming increasingly faster, and illumination design and analysis tools such as LightTools are adding statistical tolerancing capabilities. Let's look at an example that uses tolerance analysis to examine the color performance of an exterior automotive lamp that uses LEDs.

## Center High-Mounted Stop Lamp

The center high-mounted stop lamp (CHMSL), colloquially known as the "third brake light," is a signal lamp on the rear of automobiles that indicates that the vehicle is slowing or stopping when the brakes are engaged. It is located along the vehicle left/right centerline and is mounted as high as is practical, typically at the roofline or behind the rear window. CHMSLs were one of the first automotive lamps that were converted from conventional incandescent bulbs to LEDs. A CHMSL typically comprises a small set of low- to mid-power red LEDs whose light is manipulated by some combination of reflective and lenticular optics to create the required far-field intensity distribution.

For our example, we designed a 6-up linear CHMSL using TOPLED-style LEDs. These LEDs were chosen because several manufacturers make variants that should be drop-in replacements. Optically, the light from each LED is partially collimated by its own aluminized reflector and then passes through a red plastic micro-lens array, which creates the legal intensity distribution. The following figure shows the layout of the design. Analyzing the tolerances associated with keeping the intensity within the legal limits is a fascinating and complex topic all by itself, but for this article, we've chosen to concentrate on a less common but equally important consideration: the color of light emitted by the lamp.



## Working in the Red...

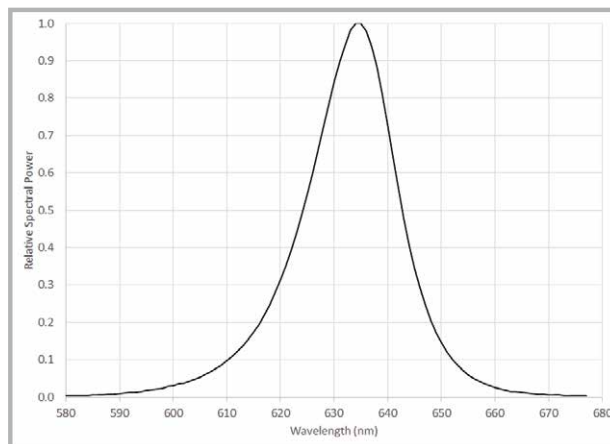
For a CHMSL to be legal, as far as the color of emitted light is concerned, its 1931 CIE chromaticity must lie within a specification-defined region known as the colorbox. Historically, this has been a very important consideration, because the light source was a warm white light bulb that was filtered by a lens to create red light. If you used the wrong thickness of lens or the wrong dye, you could very easily get a chromaticity that doesn't pass. With LEDs, it's a little bit easier, because the light from the LED is already red. For the most part, a designer just has to make sure that the optical elements in the system—the reflector and the lens—don't shift the chromaticity outside of the colorbox. In our case, this is true. Neither the spectral reflectivity of the aluminumized reflector nor the spectral transmission of the plastic lens shifts the chromaticity of the LED significantly. Therefore, for all intents and purposes, the spectrum of the CHMSL and the spectrum of the LEDs are the same. We just need to know that the LEDs we buy are within the legal region for chromaticity. Can we buy LEDs from multiple color bins? Will we have to sort the LEDs we buy, which adds to the manufacturing costs? In this article, we try to answer these important questions using tolerance analysis.

## Tolerancing Color

Tolerance analysis is a process that allows you to predict manufacturing throughput; it is accomplished by studying the effects that perturbations from the nominal design have on a pre-defined set of performance metrics. Three things are necessary to perform a tolerance analysis:

1. A set of parameters to be perturbed
2. The ranges and probability distributions associated with each perturbed parameter
3. A set of performance metrics to track during the analysis

For our CHMSL, we want to perturb the color of the light emitted by the six LEDs. Because the color of a light source in LightTools is input as the spectral power distribution, we will be perturbing the spectrum of each LED independently. We chose to fit the spectral power distribution of the nominal LED, which is provided in the LED datasheet, using an asymmetric function based on the square of a Lorentzian distribution. This function allows us to build the nominal and perturbed spectral power distributions using three parameters: the peak wavelength, the half-width at half maximum (HWHM) for the function on the side of the distribution with shorter wavelengths, and the HWHM for the function on the side of the distribution with longer wavelengths. The following figure shows the nominal spectral power distribution as obtained from the LED datasheet.

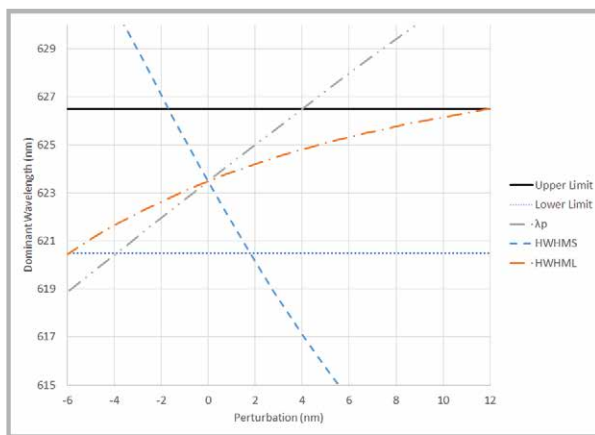


As stated, the ranges and statistics associated with perturbations to the LED spectrum are also needed. Several LED manufacturers were contacted for this information, but all of them declined to provide it. The reason for this might be very simple: they probably don't have the data. Color LEDs like these are binned on dominant wavelength, which is calculated from the chromaticity. We wouldn't measure the full spectrum when the chromaticity will do either! Lacking any real data, we settled for looking at two hypothetical scenarios: worst-case and best-guess. More on these later.

We chose to track three performance metrics: the legality of the color of the lamp, the dominant wavelength of the spectra, and the noticeability of the differences in color between the LEDs of a single lamp. The first metric, legality, is straightforward—calculate the chromaticity of the superposition of the spectra of the six LEDs in the lamp and see if it's within the colorbox. The dominant wavelength is also calculated from the chromaticity. For this metric, we want to examine the dominant wavelength for each LED individually to determine which color bins of LEDs need to be purchased. Ideally, we can accept at least one color bin of LEDs, and the worst-case would be if we must sort the LEDs we purchase ourselves. Finally, we want to look at the noticeability of the color differences between each of the LEDs in the lamp. If the lamp is too "colorful," meaning that there are too many different shades of red, then we might reject those lamps for aesthetic reasons. For this metric, we want the chromaticity of each of the LEDs to be within a 5-step du'v' circle from the nominal spectrum's chromaticity.

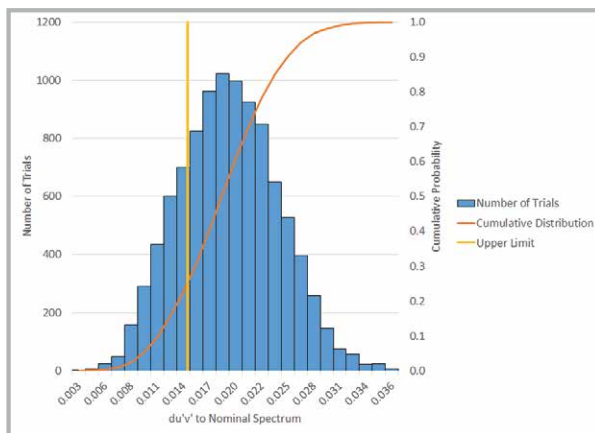
## Performance of the Worst-Case Scenario

In our hypothetical worst-case scenario, any perturbation of each of the spectral parameters is equally as likely as the nominal parameter. This translates to a uniform probability distribution for each of the spectral parameters within their respective ranges. But what are the ranges? For this, we perform a tolerance sensitivity analysis, during which we individually perturb each of the spectral parameters over an arbitrary range. From this data, we can look at the impact each spectral parameter has on our performance metrics and find a perturbation range keeps each of the metrics within their acceptable ranges. For all of the spectral parameters, the dominant wavelength performance metric requires the tightest perturbation range in order to keep our LEDs to one color bin. The following figure shows dominant wavelength as a function of perturbation for a single LED. The perturbation ranges that keep the dominant wavelength within the upper and lower limits were used for all subsequent tolerance analyses.



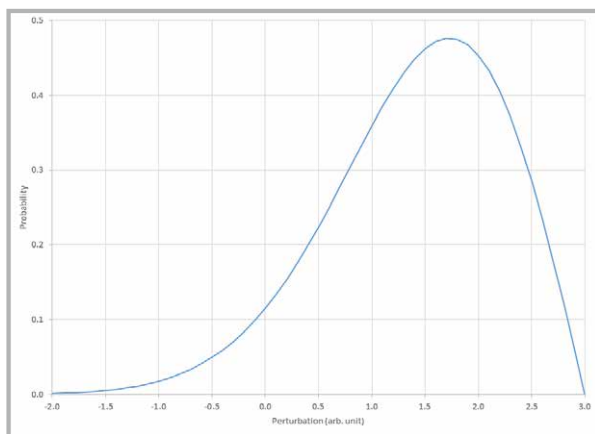
With the perturbation ranges and probability distributions defined, we can now look at the tolerances associated with an entire production run. To do this, we performed a Monte Carlo tolerance analysis, during which a large number of trials with randomly perturbed parameters was simulated. We ran 10,000 iterations for our analysis and looked at the statistics associated with each of our performance metrics.

For the legality metric, 100% of the trials resulted in a legal lamp. Great! For the dominant wavelength metric, only 67% of the trials resulted in an LED that was in our color bin. Even though we set the ranges of the spectral parameters to ensure that the dominant wavelength was within one color bin, 33% of the trials were outside that range. Why? The simple answer is cross-terms. The sensitivity analysis used to set the ranges only considers the effect of perturbing each spectral parameter individually. The Monte Carlo analysis perturbs all three spectral parameters at the same time, so the effect of multiple perturbations is considered for each trial. One solution to keep the dominant wavelength within one bin is to reduce the ranges of the spectral parameters. As for the noticeability metric, only 25% of the trials resulted in a  $du'v'$  below our acceptable level. That means 75% of the lamps produced will have at least one LED whose color is noticeably different from the others. The following figure shows the results of the Monte Carlo tolerance analysis for the noticeability metric. Most of the trials, shown by the blue histogram, fall above the upper limit for this metric.

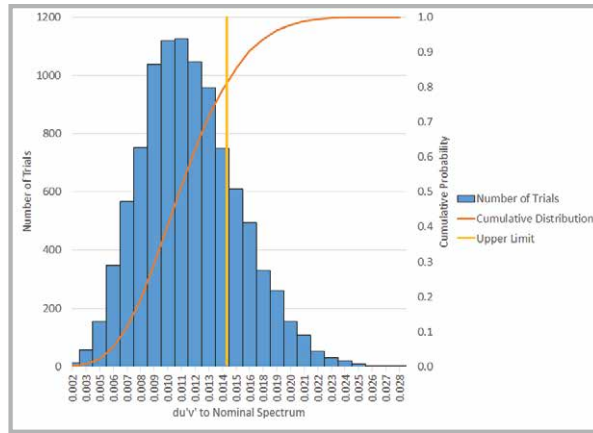


## Performance of the Best-Guess Scenario

For the hypothetical best-guess scenario, we kept the perturbation ranges the same and adjusted the probability distributions to something that “might” be more realistic. For the peak wavelength parameter, we assume that the probability distribution is normal, with the mean at the nominal value and the distribution out to  $6\sigma$  falling within the defined range. For the two HWHM spectral parameters, we assume that the value tends to broaden, and a narrower spectrum is less likely. To model this effect, we use a negative Weibull distribution, shown in the following figure, for each.



As with the worst-case scenario, we ran 10,000 iterations for the Monte Carlo analysis. As before, 100% of the trials resulted in a legal lamp. The dominant wavelength range for the trials was significantly better. 91% of the trials are now within one color bin. The noticeability metric is also better—only 20% of the lamps are more colorful than we would like. The following figure shows the results of this Monte Carlo analysis for the noticeability metric. The distribution of trials has clearly shifted so that more of the histogram area is to the left of the upper limit.



## Conclusions

Understanding the tolerances of a design is important for any system that goes into production. Tools for tolerancing illumination systems for important optical performance metrics are now making inroads and giving designers the ability to virtually prototype the bad systems as well as the nominal ones. Understanding where problems lie before going into production will reduce iterations and increase production yields. In our example, we showed how simply changing the probability distribution for a given perturbation within a fixed range can have a significant impact on the throughput in production.